Jet-Wake Thermal Characteristics of Heated Turbulent Jets in Crossflow

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This paper is one in a series reporting on jets discharging to crossflowing water streams. The experiments were carried out in the 0.61-m × 1.067-m semiclosed circuit water channel at Iowa State University. Hot water was injected vertically upward from a circular pipe located near the channel bottom to simulate the turbulent heated jet. Contours of mean and root-mean-square temperatures both across and along the jet are reported up to lateral and downstream distances of 1.15 and 37 jet diam, respectively, and for velocity ratios of 1, 2, 4, and 7. The contours were used to explain the complex interaction mechanism between the freestream and the jet in both the jet and wake regions.

Nomenclature

D = jet diameter at discharge
M = jet-to-freestream velocity ratio, U_j/U_∞
T = mean temperature
T_j = jet discharge temperature
T_∞ = freestream temperature
t' = fluctuating temperature
U_j = jet discharge velocity
U_∞ = freestream velocity
X = streamwise distance
Y = transverse (spanwise) distance
Z = vertical distance

Introduction

In most practical situations, jets and plumes are either discharged vertically or at an angle to a crossflow. This is the case in exhaust gases discharging into the atmosphere, condenser cooling water pouring into rivers, and the jet in the V/STOL aircraft.

The words jet and plume are sometimes used interchangeably. A plume is, in general, a free shear flow in which the primary source of kinetic energy and momentum flux arises through body forces. In a jet the kinetic energy and momentum are large at discharge, having been imparted to the flow by a pressure drop through an orifice, nozzle, or tube. A forced plume or a buoyant jet are flows whose motion is in transition from a jet to a plume.[1]

Experimental work on the behavior of plumes in flowing ambients is numerous. Bryant and Cowdrey[2] studied the trajectory of heated air plumes in a wind tunnel. Slawson and Csanady[3] conducted experiments to determine the mean path of buoyant bent-over chimney plumes. Halitsky[4] visualized smoke plumes with a single camera. Hewett et al.[5] and Hewett[6] studied smokestack plumes in a stable atmosphere. Bringfelt[7] measured plume rise at industrial chimneys. The same problem was also studied by Leavitt et al.,[8] Carpenter et al.,[9] and Thomas et al.[10] Weil[11] and Barilla[12] injected ferric chloride into a water towing tank in order to model a plume in a laminar crossflow. Barilla varied the initial angle of the issuing plume and the bending angle of the discharge nozzle for different Reynolds numbers and observed that far downstream of the discharge nozzle the plume rise is independent of the orientation of the discharge.

Experiments on round turbulent nonbuoyant jets where the main thrust was on velocity and/or turbulence intensity measurements were conducted by Jordinson,[13] Shandorov,[14] Gordier,[15] Keffer,[16] Keffer and Baines,[17] Pratte,[18] Pratte and Baines,[19] Gerend,[20] Patrick,[21] Platten and Keffer,[22] Margason,[23] Bergeles et al.,[24] Storms,[25] Gelb and Martin,[26,27] Crowe and Riesebieter,[28] Chassaing et al.,[29] and Chan and Kennedy.[30]

Experiments on round turbulent buoyant jets in which the main thrust was on velocity, concentration, temperature, or trajectory measurements were carried out by Fan,[31,32] Abraham and Eysink,[33] Abraham,[34] Turner,[35] Vadot,[36] Ayoub,[37] Cavola,[38] and Cavola and Davis.[39]

Experiments on round turbulent heated nonbuoyant jets in which the main thrust was on trajectory, velocity, temperature, or penetration measurements were carried out by Callaghan and Ruggeri,[40,41] Callaghan and Bowden,[42] Ruggeri and Callaghan,[43] Ruggeri,[44] Campbell and Schetz,[45-47] Payne and Schetz,[48] Ramsey,[49] Ramsey and Goldstein,[50] Kamotani and Greber,[51] and Wark and Foss.[52]

Experiments on turbulent nonbuoyant jets in which the main thrust was on studying the structure of the jet wake, jet cross section, vortex shedding in the jet, or measuring the vorticity in the jet or wake were carried out by Chang-Lu and Hsiu-Chen,[53] Rouse,[54] McAllister,[55] Pratte and Keffer,[56] McMahon et al.,[57] Fearn and Weston,[58] Weston,[59] Moussa,[60] and Moussa et al.[61]

Experiments on round turbulent nonbuoyant jets in which the main thrust was on pressure measurements were carried out by Vogler,[62] Peake,[63] Bradbury and Wood,[64] Wooler et al.,[65] Hackett and Miller,[66] McMahon and Mosher,[67] Mosher,[68] Wu et al.,[69] Mikolowsky,[70] and Mikolowsky and McMahon.[71]

Measurements on round or plane, heated or unheated, turbulent jets in which the main emphasis was on studying the turbulence structure in terms of Reynolds stresses, turbulent heat flux, or higher-order moments were carried out by

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Crabb,[72] Crabb et al.[73] Andreopoulos,[74-76] Andreopoulos and Rodi,[77] Ramaprian and Haniu,[78] Wu et al.,[79] and Sherif and Pletcher.[80,81]

Although the measurements reported in the literature seem to be numerous, few studies went beyond investigating the jet trajectory and spread. Measurements dealing with the mean temperature field are generally fewer than those on the velocity field. With the exception of the work by Andreopoulos,[74] Ramaprian and Haniu[78] (two-dimensional jets), and Sherif and Pletcher,[81] there is no work known to the authors that reported measurements on the root-mean-square temperature fluctuations. The measurements reported by Andreopoulos,[74] however, are applicable to low velocity ratio jets (less than 2.0).

Mechanism of Interaction Between a Jet and a Cross Section

According to measurements and flow visualization studies conducted by previous investigators, a fairly detailed description of the jet can be presented. When a jet is discharged vertically into a crossflowing stream, three distinct regions can be distinguished.

1) The initial region, or the zone of flow establishment (also called the zone of residual inlet velocity),[29] where the initially uniform jet flow interacts with the ambient crossflow causing a shear layer to develop at the jet boundaries. A cone-shaped cross section in this zone is formed where the turbulence intensity is relatively small, the velocity is nearly uniform, and the flow is nearly irrotational.[30] Because of a slight jet curvature in this zone, the cone-shaped potential core tends to deflect (see Fig. 1). At the upstream region of the jet, the flow is decelerated and a positive pressure region is formed.[16] Severe lateral shear stresses, directed toward the jet wake, act on the jet sides. A region of low pressure is formed in the jet wake, and separation there ultimately occurs. The length of the jet potential core is generally a function of the jet diameter and velocity ratio. Pratte and Baines[19] argue that it is also a function of the jet discharge Reynolds number. Keffer and Baines[17] also confirm this observation, but no functional trend is reported in the literature for this Reynolds number dependency.

2) The main region, or the zone of established flow (also called the zone of accommodation[29] and the curvilinear zone).[30] In this region a turbulent mixing layer develops around the jet boundaries and the flow quickly becomes fully turbulent.[82] The jet sides experience strong lateral deflections because of the shearing action of the cross stream. The potential core rapidly erodes primarily because of the growth of the mixing layer along the jet flow direction. Keffer[82] reports that the length of the potential core is a direct function of the velocity ratio and is of the order of 1, 2, and 3 orifice diameters for velocity ratios of 4, 6, and 8, respectively.

Jordinson[13] observed that the flow in this region is similar to that over a porous cylinder with suction. He reports that, in the jet wake, the main stream fluid is drawn into the cylinder so that the angles of the flow to the main stream direction are greater than 90 deg. The analogy given by Jordinson is probably not complete and does not adequately describe the entrainment mechanism at the jet front and in the wake. A pair of vortices is formed in the wake region which expands and intensifies by receiving vorticity produced by the interaction between the freestream and the jet.[30] The jet cross-sectional shape is changed to a characteristic kidney shape by the action of the pressure and lateral shear forces. The jet sides are easily deflected since they possess less momentum than the jet center because of the lateral mixing process.[17] The result is a pair of counter-rotating swirls, the strength of which reaches a maximum at some point downstream from the potential core region.[30] The helicity in the wake region contributes greatly to the mechanism by which the freestream fluid is entrained into the jet fluid.[19]

3) The far-field region (also known as the zone of velocity profile similarity)[29] where the magnitude and direction of the jet velocity are close to those of the crossflowing stream, and it becomes more and more difficult to distinguish freestream and jet fluids. The jet, however, will continue to rise, partly because the vertical component of the jet original momentum is preserved.[82] The rate of rise, however, will continue to decrease primarily because the diffusion of the jet momentum takes place over a continuously increasing area. The two vortices will continue to move at nearly the velocity of the freestream, their strength will decrease, and their size will continue to increase.[30,58,59] The decrease in the amount of circulation is partly attributed to the increase in the rate of ordinary viscous dissipation. Keffer[82] reports that the jet turbulence structure in this region is dominated by the viscous decay. He argues that the vortices in this phase should be viewed as general circulation patterns rather than discrete line vortices. The measurements of Pratte and Baines[19] indicate that the entrainment starts to decrease and that the entrainment mechanism becomes entirely dependent on the vortex flow.

Results of more recent investigations[73,75-77] at low velocity ratios indicate that the upstream flow in the jet dischage pipe is influenced by the crossflowing stream. For example, Crabb et al.[73] report that the distortion in the velocity profile at the pipe exit is more pronounced at a velocity ratio of 1.15 than at 2.3. For both ratios, however, the front half of the jet in the plane of symmetry has a decreased velocity, and the rear half is forced to accelerate, and probably widen, to compensate for the extra flow of jet fluid. Crabb et al. report that this rear-edge acceleration is more noticeable in the lower velocity case.

This paper reports on mean and fluctuating temperature measurements in a heated nonbuoyant jet for velocity ratios of 1, 2, 4, and 7. The measurements are presented in the form of contours of mean and fluctuating temperatures both across and parallel to the jet. The jet and wake cross sections are revealed, showing regions of low and high temperature fluctuations and their relationship to the flow. Throughout the paper, reference is made to other work describing the jet-wake interaction in terms of other parameters such as pressure, velocity, turbulence, and temperature.

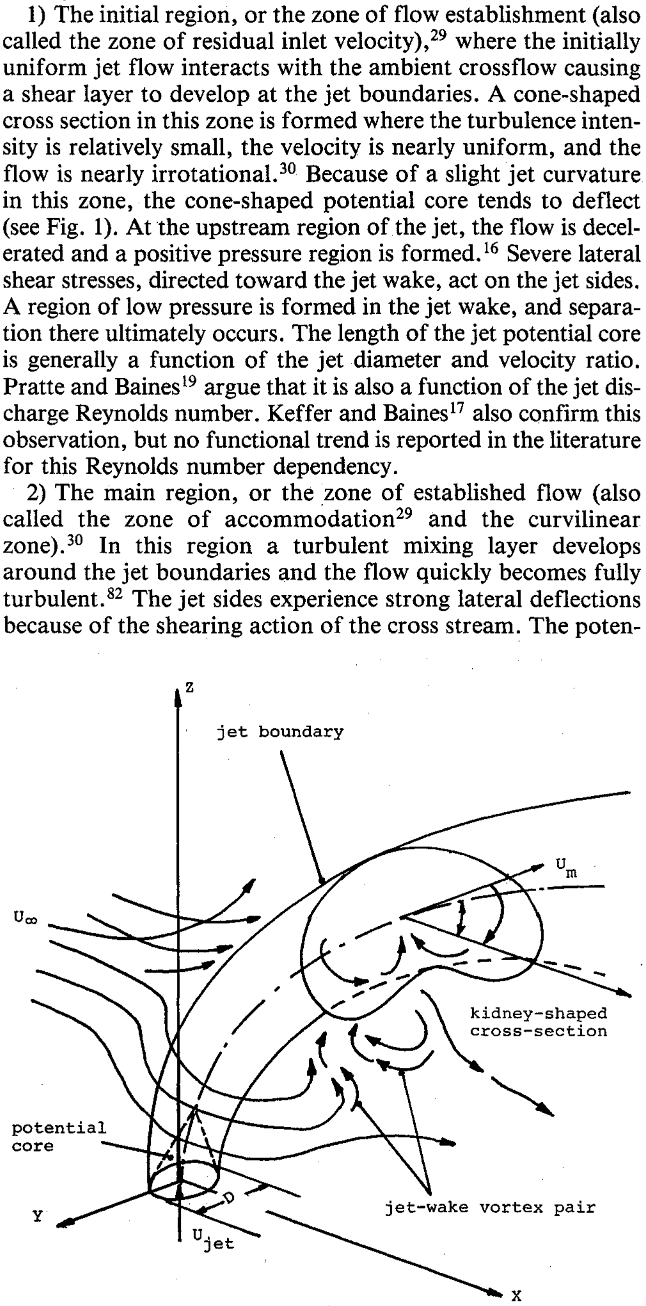

Fig. 1 Round turbulent jet in a crossflow.

Experimental Program

As mentioned earlier, the measurements were carried out in the 0.61-m × 1.067-m semiclosed circuit water channel at Iowa State University. The test section was formed by an aluminum plate, 12.7 mm thick, 0.457 m wide, and 1.829 m long with a rounded (1:2 ellipse) leading-edge and a trailing-edge flap. The plate was positioned 152.4 mm above the bottom of

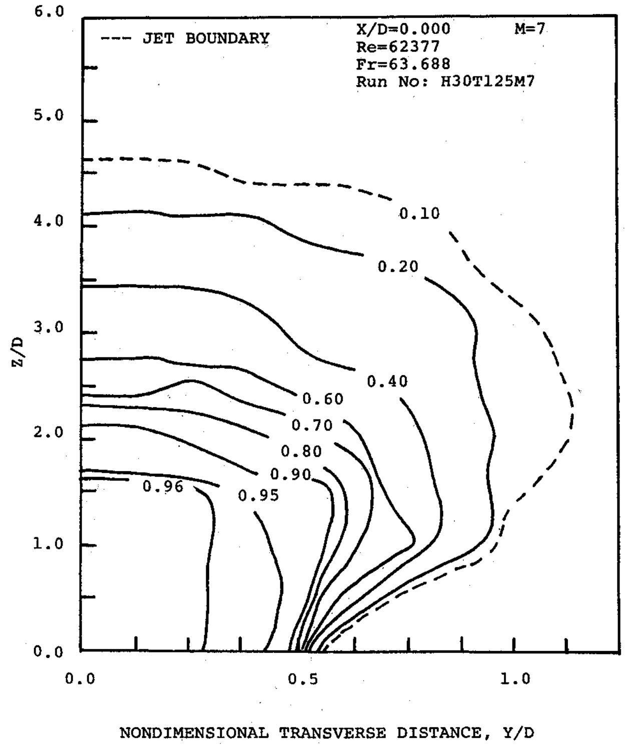

Fig. 2a **Contour plots of $(T - T_\infty)/(T_j - T_\infty)$, $M = 7$, $X/D = 0$.**

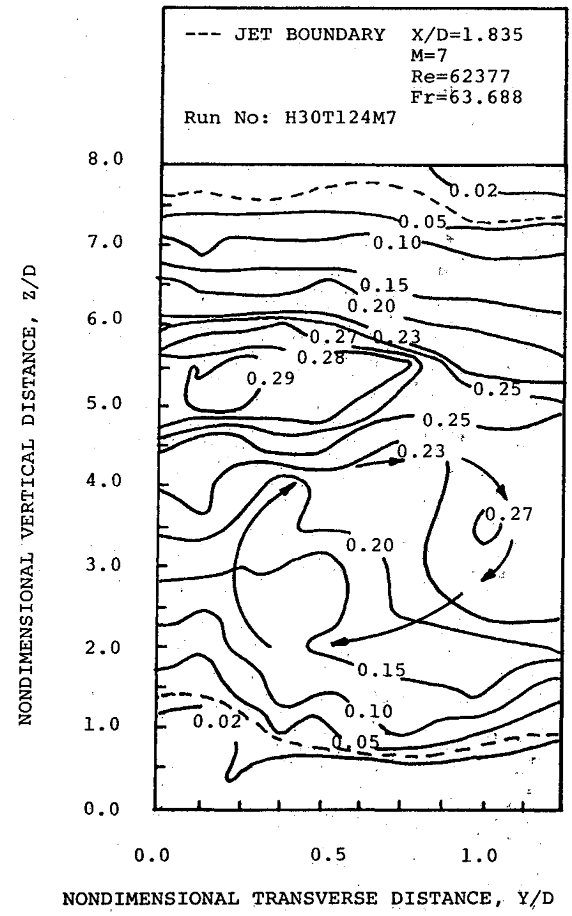

Fig. 2b **Contour plots of $(T - T_\infty)/(T_j - T_\infty)$, $M = 7$, $X/D =$ 1.835.**

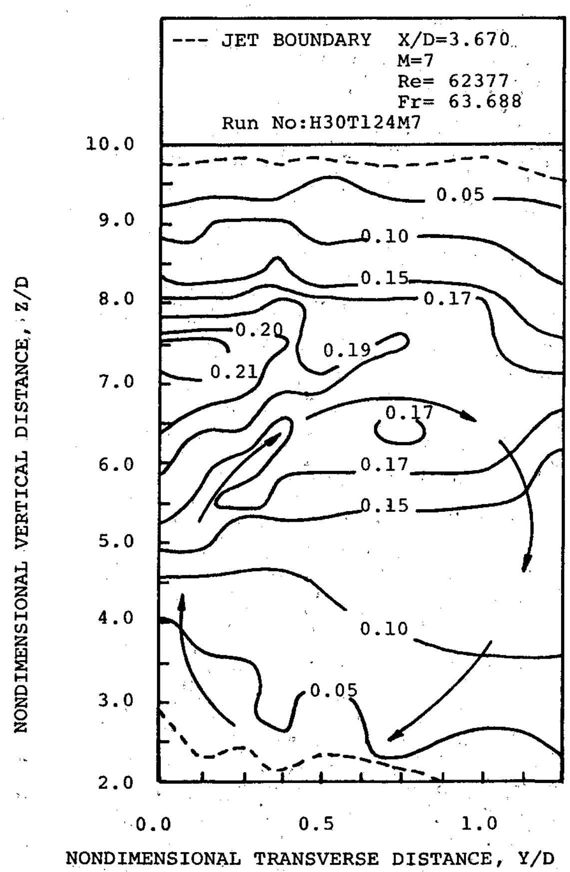

Fig. 2c **Contour plots of $(T - T_\infty)/(T_j - T_\infty)$, $M = 7$, $X/D =$ 3.670.**

the channel. The plate length was chosen taking into account the range of jet trajectories for different jet inlet conditions. The plate width was chosen so as not to interfere with the turbulent boundary layers already developed on the channel sides. Two false sidewalls having a thickness of 8.73 mm, a height of 0.965 m, and the same plate length of 1.829 m were installed on both sides of the plate to promote two dimensionality for the flow and to serve as supporting members for the plate. Hot water was injected vertically upward from the plate through a circular pipe of 13.84 mm i.d. The thickness of the boundary layers developing on the false walls has been kept to a minimum by providing streamlined leading edges to the false walls. The flap provided at the plate trailing edge was adjusted so as to minimize the elliptic effects in the flowfield and to ensure that the flow approached the plate at a zero angle of incidence. A DISA constant current anemometer was used in conjunction with a DISA 55R11 fiber-film probe for mean and fluctuating temperature measurements. The constant-current anemometer was comprised of a DISA 55M01 main unit, a DISA 55M05 power pack, and a DISA 55M20 temperature bridge. A constant probe current of 2.5 mA was used for all runs. This relatively high value of the probe current was chosen because of the desire to increase probe sensitivity to temperature fluctuations. The velocity dependence was checked and was found to be negligible at this value of the probe sensor current. The DISA 55R11 probe was calibrated for temperature measurements using a Haake A81 constant temperature bath with a resolution of 0.056°C and a calibrated copper-constantan thermocouple attached to the fiber-film probe support. Additional details about the calibration method, experimental facility, data acquisition system, and measurement techniques can be found in Sherif and Pletcher.[80,81] The overall uncertainties in the measurement of the mean temperature and root-mean-square temperature fluctuations were less than 2 and 12%, respectively.

Qualification of the measurement techniques was done in part by comparisons with mean temperature data of those of Ramsey and Goldstein,[50] Kamotani and Greber,[51] and Andreopoulos.[74] Other qualification tests can be found in Sherif and Pletcher.[80]

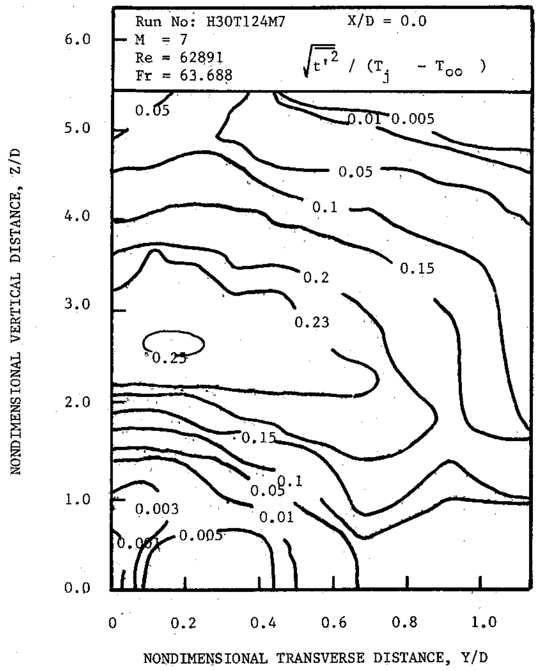

Fig. 3a Contour plots of $\sqrt{\overline{t'^2}}\,/\,(T_j - T_\infty)$, $M = 7$, $X/D = 0$.

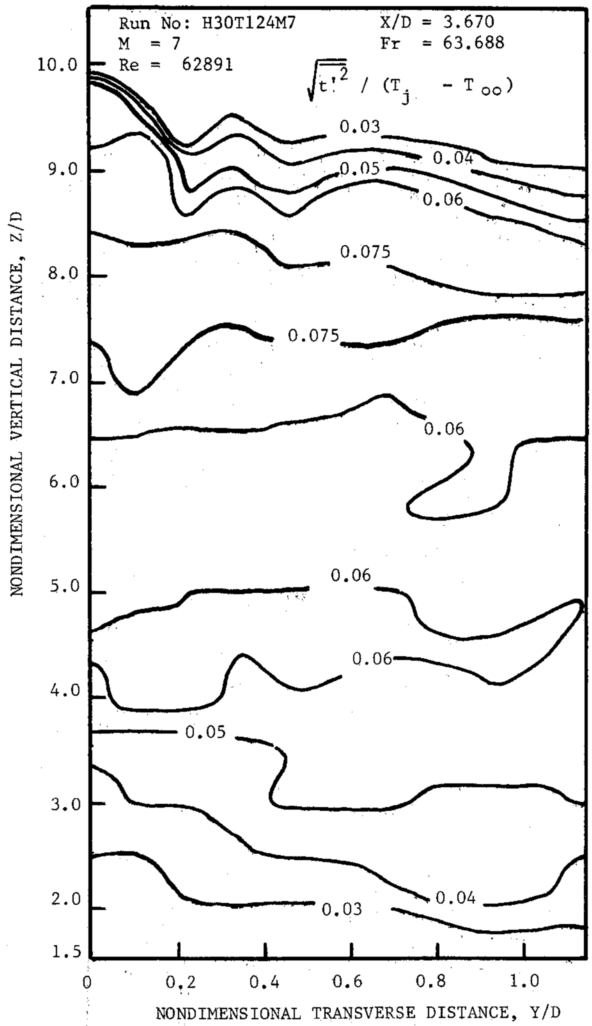

Fig. 3b Contour plots of $\sqrt{\overline{t'^2}}\,/\,(T_j - T_\infty)$, $M = 7$, $X/D = 3.67$.

Results and Discussion

As mentioned earlier, the jet/wake thermal characteristics will be presented in terms of contours of mean and fluctuating

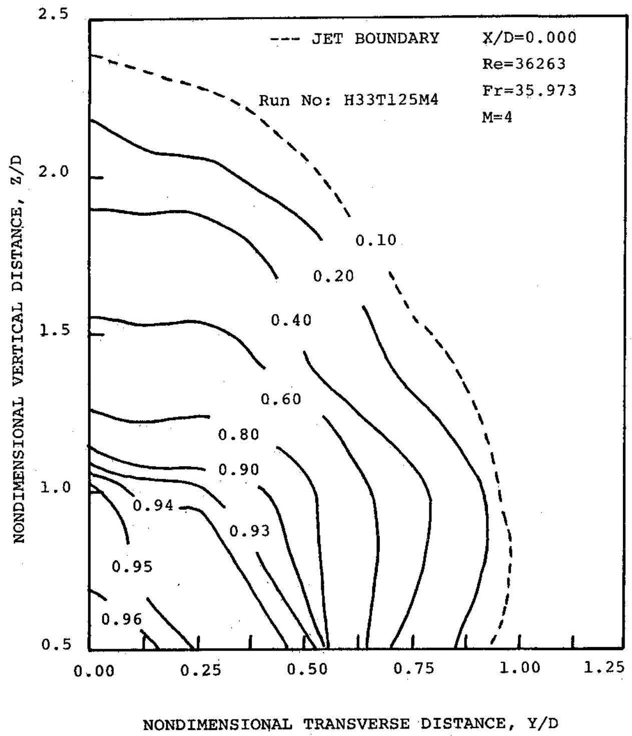

Fig. 4 Contour plots of $(T - T_\infty)/(T_j - T_\infty)$, $M = 4$, $X/D = 0$.

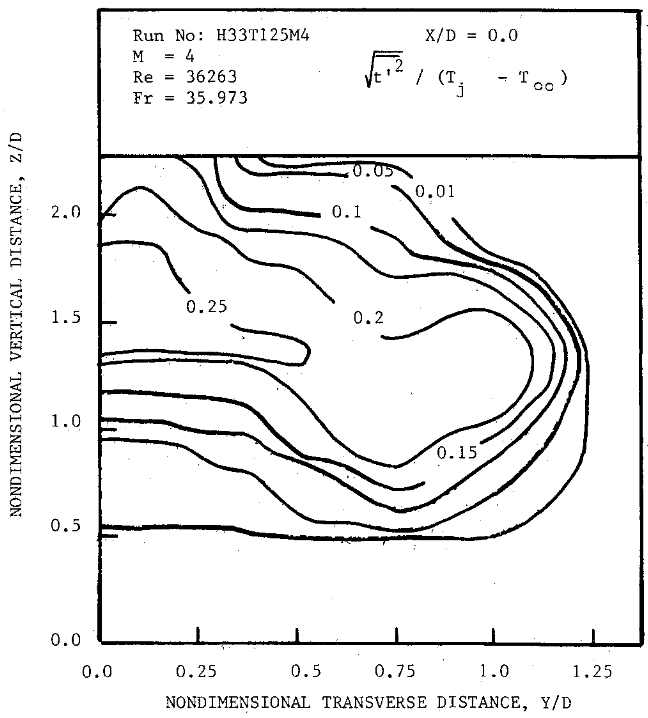

Fig. 5 Contour plots of $\sqrt{\overline{t'^2}}\,/\,(T_j - T_\infty)$, $M = 4$, $X/D = 0$.

temperatures both across and parallel to the jet. In a previous paper,[81] the authors examined mean and fluctuating temperature profiles for velocity ratios of 1, 4, and 7. In that paper it was established that the $M = 1$ jet behaved distinctly differently than the $M = 4$ and $M = 7$ jets. In order to examine this phenomenon more closely, the authors decided to study the $M = 2$ jet. This paper reports in part on that.

For $M = 7$, isotherms in cross planes (perpendicular to the jet plane of symmetry) are presented in Figs. 2a, 2b, and 2c for downstream positions of $X/D = 0$, 1.835 and 3.670, respectively.

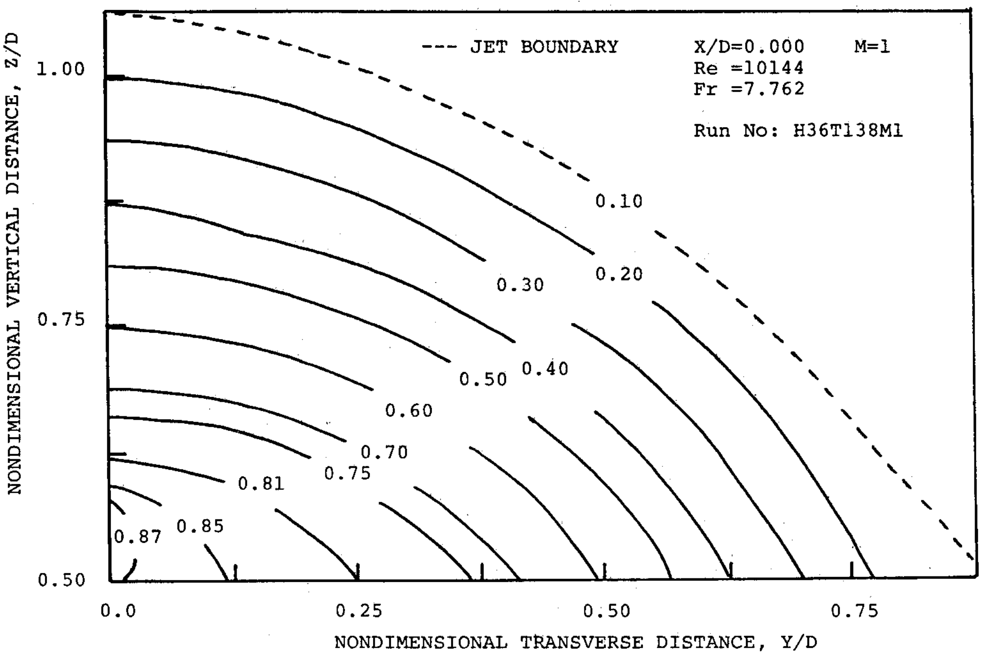

Fig. 6a Contour plots of $(T - T_\infty)/(T_j - T_\infty)$, $M = 1$, $X/D = 0$.

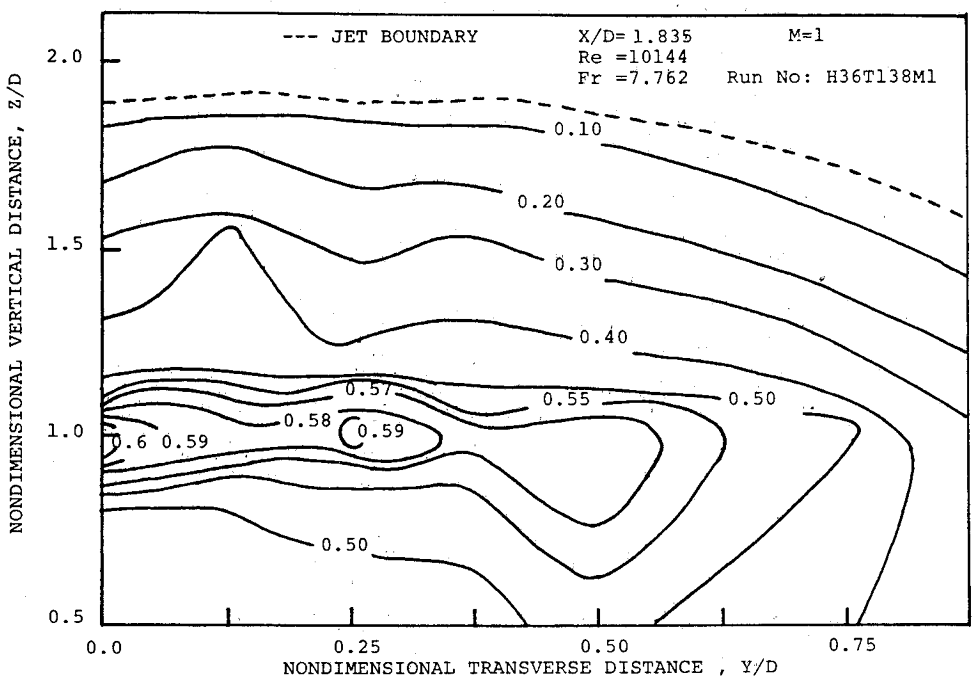

Fig. 6b Contour plots of $(T - T_\infty)/(T_j - T_\infty)$, $M = 1$, $X/D = 1.835$.

Above the jet discharge plane $X/D = 0$ (see Fig. 2a) the isotherms appear as more or less concentric circles wih the highest temperature being close to the center. Slightly farther downstream, at $X/D = 1.835$ (see Fig. 2b), the isotherms appear slightly distorted. The highest temperature is shown to correspond to a vertical position of $Z/D = 5.25$. This was shown earlier[81] to be the location of the jet temperature centerline trajectory. Below $Z/D = 4.3$, the isotherms look more nonuniform, indicating a region of more activity (mixing). According to Keffer,[82] the zone of flow establishment should end around $X/D = 2.5$ for this velocity ratio. The isotherms displayed in Fig. 2b are therefore representative of the jet/wake thermal behavior in this zone.

At a downstream position of $X/D = 3.67$ (see Fig. 2c), more nonuniformities in the isotherms are noticeable. The jet centerline is approximately located at a vertical distance of $Z/D = 7.4$ with a local temperature excess ratio about 21% of that at the discharge. This downstream position is about 1 diam into the second zone (zone of established flow) according to Keffer.[82]

In all three figures, dashed lines indicating the location of the jet upper and lower edges are shown. The jet edge as presented here is based on a concept similar to that introduced by Squire.[83] According to Squire, the jet edge is defined as the boundary on which the velocity is 10% of the centerline value. In the present investigation, a similar definition was adopted employing the temperature excess ratio in place of velocity. The jet edge as introduced here, however, should be taken to imply both the jet and its wake. This implies that within the "jet boundaries" most of the activities will take place and that

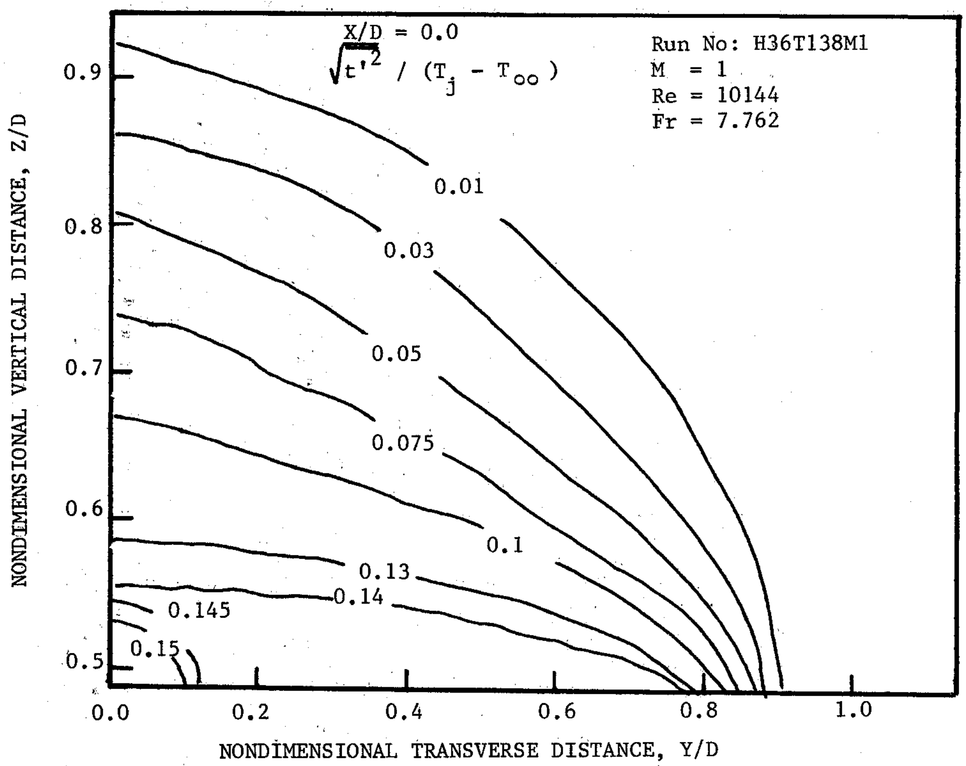

Fig. 7a Contour plots of $\sqrt{t'^2} / (T_j - T_\infty)$, $M = 1$, $Y/D = 0$.

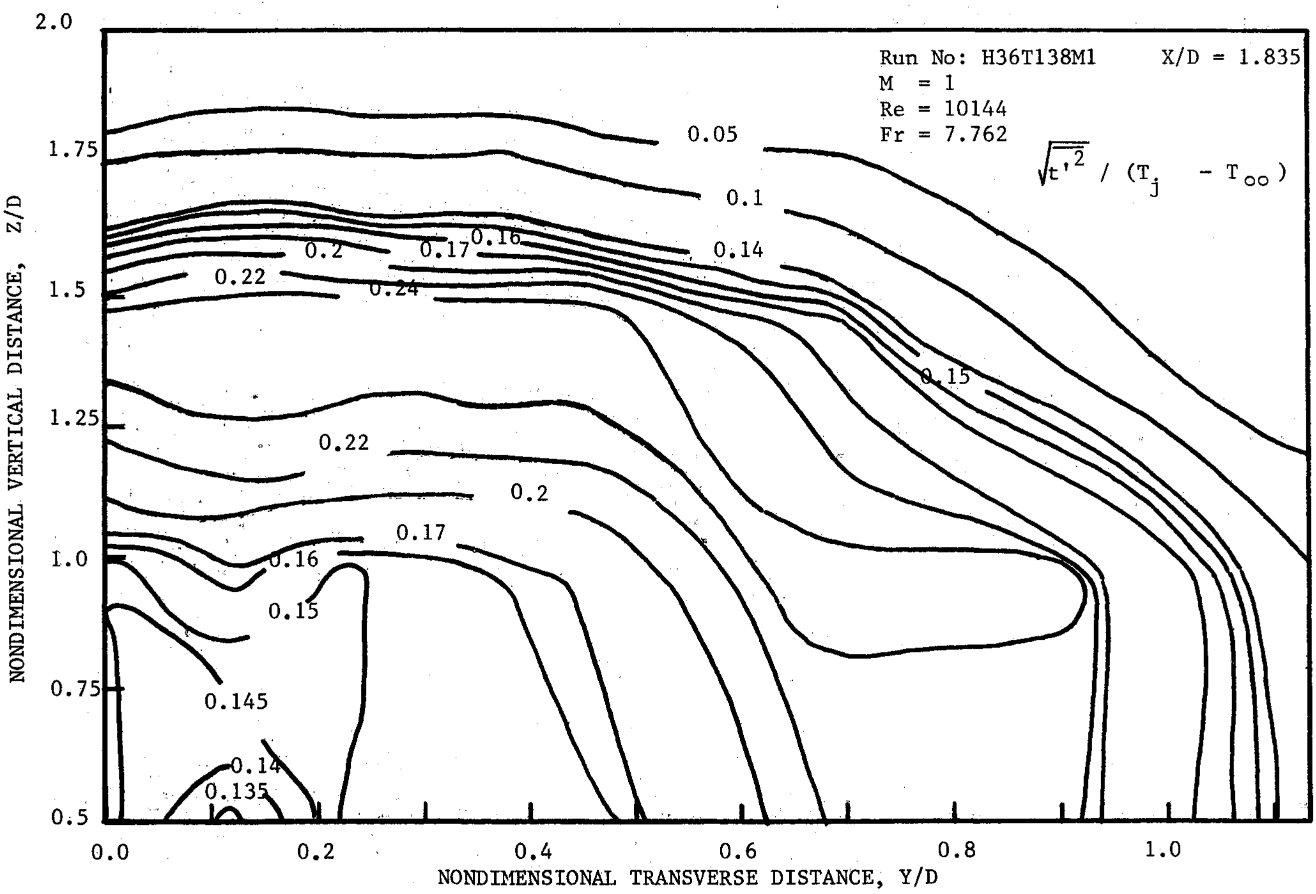

Fig. 7b Contour plots of $\sqrt{t'^2} / (T_j - T_\infty)$, $M = 1$, $X/D = 1.835$.

the regions not included within the boundaries are mostly free-stream or low-activity regions. It will be shown a little later, however, that that may not be quite true for a low velocity ratio jet.

The temperature fluctuation contours for the same velocity ratio and two representative downstream positions are presented in Figs. 3a and 3b, respectively. Additional contours of temperature fluctuations can be found in Sherif.[84]

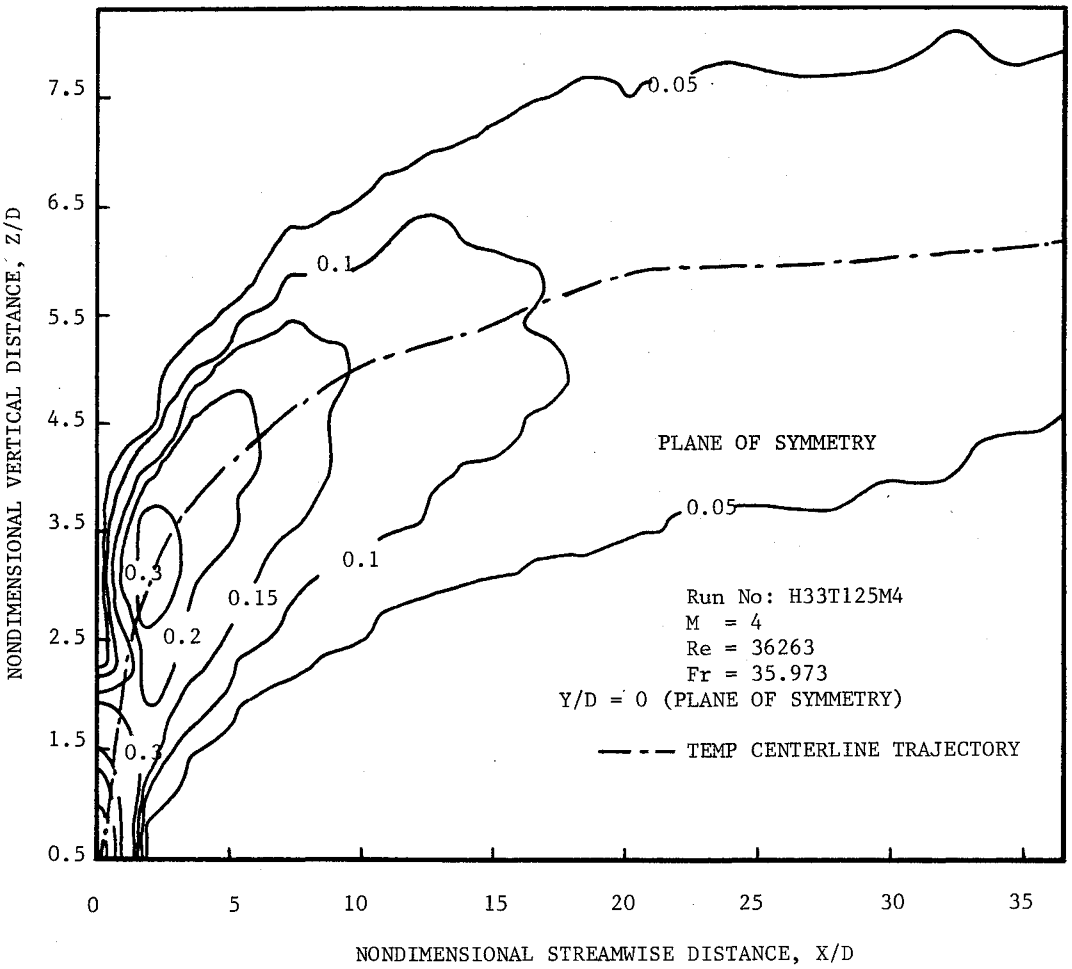

Fig. 8 Contour plots of $(T - T_\infty)/(T_j - T_\infty)$, $M = 4$, $Y/D = 0$.

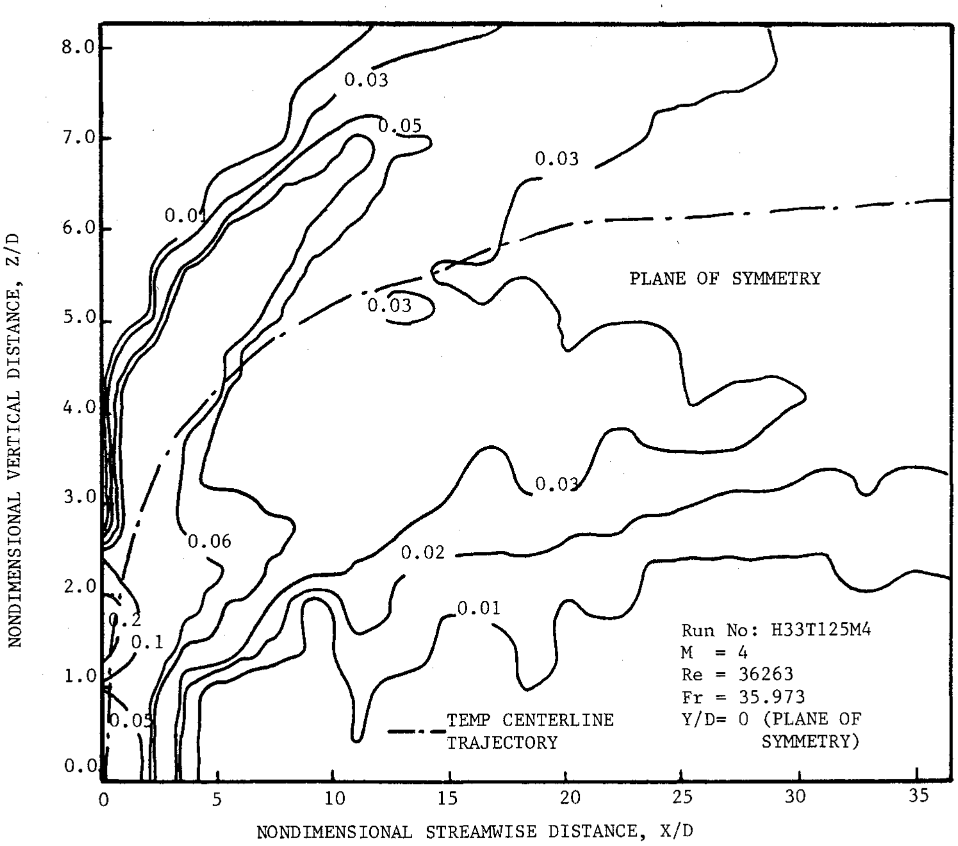

Fig. 9 Contour plots of $\sqrt{\overline{t'^2}}\,/\,(T_j - T_\infty)$, $M = 4$, $Y/D = 0$.

Examination of Figs. 3a and 3b indicates that the fluctuating temperature exhibits a rather different behavior than the mean temperature. Above the jet discharge plane ($X/D = 0$), fluctuations reach a maximum at about a vertical position of 2.6. Referring to Fig. 2a, it becomes apparent that the maximum fluctuations occur above the jet centerline in a region of high mean temperature gradient. Referring to Fig. 2c, it is easily seen that the mean temperature excess ratio drops from 0.9 to 0.6 when the vertical distance is increased from 2.2 to 2.75, respectively. This results in a temperature gradient of 0.546. Similarly, between $Z/D = 2.75$ and $Z/D = 4.6$, the temperature excess ratio drops from 0.6 to 0.1, resulting in a gradient of 0.27, which is about half the temperature gradient immediately above the jet discharge. The results are also consistent with the findings of Keffer,[82] who observed that the turbulence intensity in the potential core is small. For example, for vertical distances Z/D smaller than 0.5, a temperature fluctuation of only 0.1% of the temperature excess at the discharge is observed (see Fig. 3a). This compares with a maximum value of 25% at about Z/D of 2.75. This implies that the temperature fluctuation behaves rigorously in the same manner as the turbulence intensity. It may not be inaccurate therefore to refer to the former as a turbulent temperature.

The authors[81] have established that the fluctuating temperature (like many other turbulent quantities)[80] exhibits a double-peak pattern. The lower peak is smaller and usually occurs in the wake, whereas the higher peak is larger and occurs at a region of maximum temperature gradient above the jet centerline. This behavior is rather difficult to see by merely examining the contour plots and is best illustrated by examining the fluctuating temperature profiles.

At a velocity ratio of $M = 4$, a more or less similar behavior is exhibited by the jet. Mean and fluctuating temperature contours for $X/D = 0$ are presented in Figs. 4 and 5, respectively. Additional contours at $M = 4$ can be found in Sherif and Pletcher.[81]

At a velocity ratio of 1, the jet appears to exhibit a different behavior. This can be demonstrated by examining contours of mean and fluctuating temperature at $X/D = 0$ and 1.835 in Figs. 6a and 6b, and 7a and 7b, respectively. Contours at positions farther downstream can be found in Sherif.[84]

Immediately above the jet discharge plane ($X/D = 0$), both mean and fluctuating temperature contours are remarkably uniform (Figs. 6a and 7a). Farther downstream at $X/D =$ 1.835, the jet centerline rises as indicated by relatively large temperature excess ratios at higher vertical positions. For ex-

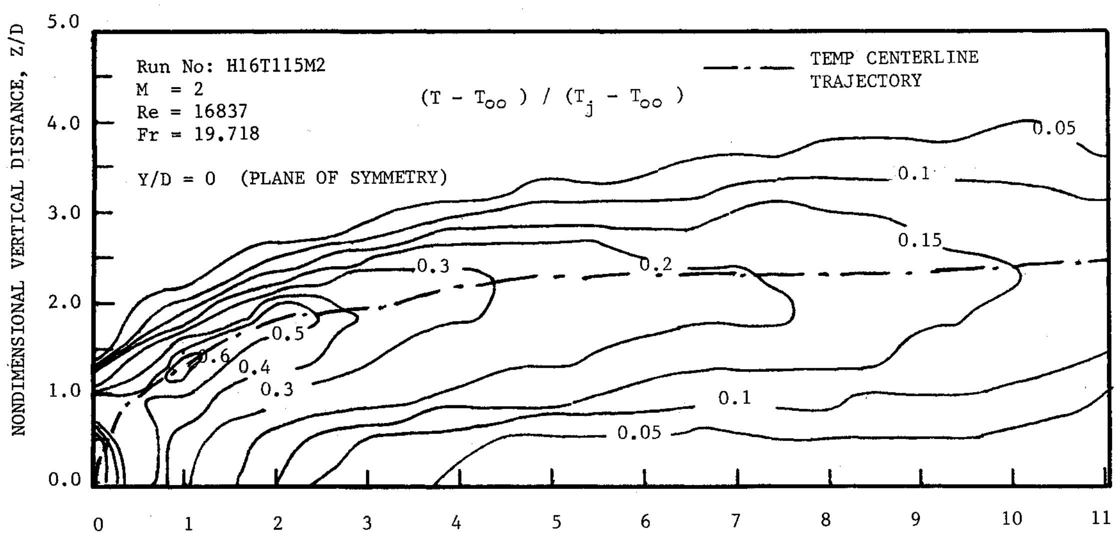

Fig. 10 Contour plots of $(T - T_\infty)/(T_j - T_\infty)$, $M = 2$, $Y/D = 0$ (plane of symmetry).

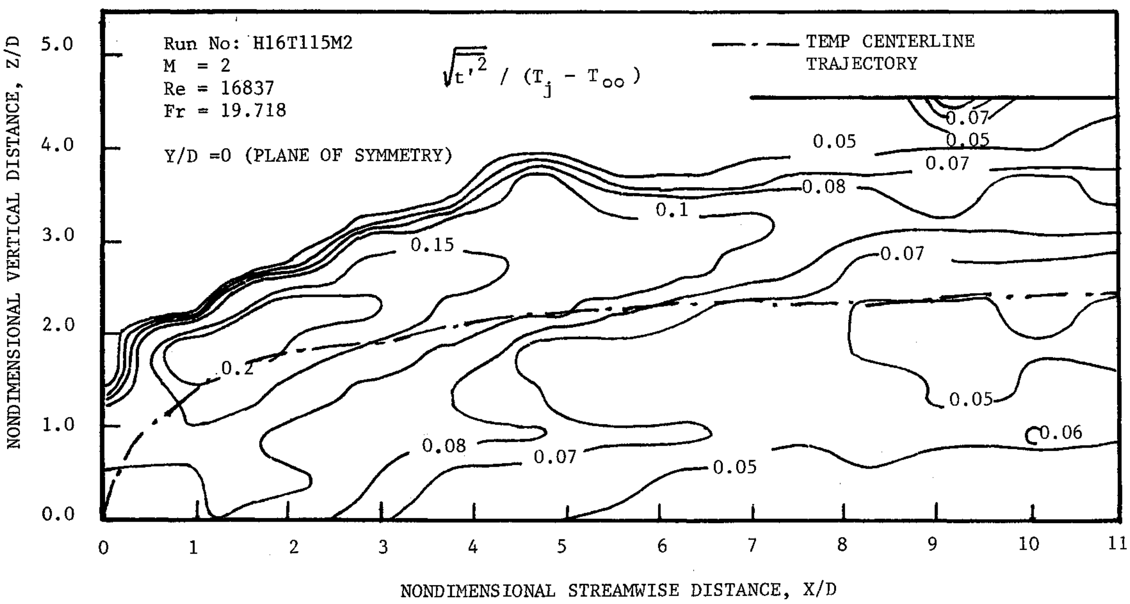

Fig. 11 Contour plots of $\sqrt{\overline{t'^2}}\,/\,(T_j - T_\infty)$, $M = 2$, $Y/D =$ (plane of symmetry).

ample, Fig. 6b indicates a maximum temperature excess ratio in the jet plane of symmetry ($Y/D = 0$) of 0.6 at about a Z/D of 1. The contours in Fig. 6b in the jet core look more or less like a kidney, confirming the existence of the well-known kidney-shaped region. It is interesting to note, however, that the wake region is very small (almost nonexistent) as evidenced by high values of the temperature excess ratio near the jet discharge plane ($Z/D = 0$). The same trend continues at positions farther downstream (see Sherif[84]). Above the jet centerline, the contours exhibit a uniform appearance indicating a region of freestream fluid and minimum mixing.

The fact that the wake region is small for this velocity ratio suggests that freestream fluid is not sucked underneath the jet as it would be in a larger velocity ratio jet. As a consequence, it is believed that the entrainment mechanism is not as efficient in the former case and that the fluid underneath the jet is mostly jet fluid (not freestream). These observations are reaffirmed by examining the temperature fluctuation data presented in Fig. 7b for the same jet.

The small velocity ratio jet is further different in terms of the double-peak pattern for the turbulent quantities.[80,81] For the fluctuating temperature, one can observe only one peak that occurs in a region of high temperature gradient above the centerline temperature trajectory.

The far-field zone is best illustrated through examination of contour plots in the jet plane of symmetry. This is done for velocity ratios of 4 and 2 in Figs. 8 or 9 for $M = 4$ and Figs. 10 and 11 for $M = 2$.

Figures 8 and 10 for the mean temperature contours reveal an interesting fact about the jet in crossflow. It is obvious that this type of flow is asymmetric with the jet temperature axis being curved toward the lower boundary of the jet. Previous investigators have observed this phenomenon and attributed it to the presence of a pressure field associated with the obstacle formed by the jet near the discharge. This pressure field is characterized by a low pressure region upstream of the jet front boundaries (see Chassaing et al.[29]).

The fluctuating temperature contours in the plane of symmetry, on the other hand, reveal the regions of high and low fluctuations up to $X/D = 37$. It is easy to see that the region around the centerline trajectory is indeed a region of low fluctuations. For example, for $M = 4$, the region of maximum fluctuations is mostly bounded by a Z/D of 1.2 and 2 and X/D of 0 and 1.5. In this region, fluctuations on the order of 20–23% of the temperature excess at discharge are observed. About 15 diam downstream close to the jet centerline, fluctuations on the order of 3% are seen. Still, the effect of the jet can be felt as far as 37 jet diam downstream. Ades[85] observed a jet as far as 100 diam downstream.

The fluctuation contours are most uniform around the jet centerline. Away from the centerline and particularly in the wake region, the contours are most nonuniform. The magnitude continues to decrease with increasing downstream distance primarily because of heat transferred to the freestream fluid.

The contour plots presented in Figs. 10 and 11 for $M = 2$ reveal some interesting observations. Like the $M = 1$ jet, the wake region is small as relatively high temperatures are observed near the solid boundary. It is not inaccurate to assume that the $M = 2$ jet is a borderline case between the low and high velocity ratio jets. In other words, one should expect the $M = 2$ jet to exhibit characteristics similar to both the $M = 4$ and $M = 1$ jets. The validity of this assertion can be checked by examining Figs. 10 and 11 and comparing them with the corresponding figures for $M = 4$ and $M = 1$.

Conclusions

A fairly detailed picture of the jet-wake thermal characteristics is presented for different velocity ratios and downstream positions. The behavior of the jet was described by examining contours of mean and fluctuating temperatures both across and parallel to the jet plane of symmetry. It was established that a small velocity ratio jet behaves differently than a large velocity ratio jet ($M > 2$). The existence of a double vortex structure was confirmed in both jets but was found to be weaker in a small velocity ratio jet. Thus, it was concluded that the entrainment and mixing mechanisms are less effective in the latter case. The $M = 2$ jet is believed to be a borderline case between the two jets.

Acknowledgments

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